

Q.11.1: Find the:

- (a) Maximum frequency, and  
(b) The minimum wavelength of X-rays produced by 30 kV electrons.

Ans:

Electron potential,  $V = 30 \text{ kV} = 3 \times 10^4 \text{ V}$

Hence, electron energy,  $E = 3 \times 10^4 \text{ eV}$

Where,  $e =$  Charge on one electron  $= 1.6 \times 10^{-19} \text{ C}$

(a) Maximum frequency by the X-rays  $= \nu$

The energy of the electrons:

$$E = h\nu$$

Where,

$h =$  Planck's constant  $= 6.626 \times 10^{-34} \text{ Js}$

$$\text{Therefore, } \nu = \frac{E}{h}$$

$$= \frac{1.6 \times 10^{-19} \times 3 \times 10^4}{6.626 \times 10^{-34}} = 7.24 \times 10^{18} \text{ Hz}$$

Hence,  $7.24 \times 10^{18} \text{ Hz}$  is the maximum frequency of the X-rays.

(b) The minimum wavelength produced:

$$\lambda = \frac{c}{\nu}$$

$$= \frac{3 \times 10^8}{7.24 \times 10^{18}} = 4.14 \times 10^{-11} \text{ m} = 0.0414 \text{ nm}$$

Q.11.2: The work function of caesium metal is 2.14 eV. When light of frequency  $6 \times 10^{14} \text{ Hz}$  is incident on the metal surface, photoemission of electrons occurs. What is the

- (a) maximum kinetic energy of the emitted electrons,  
(b) Stopping potential,  
(c) maximum speed of the emitted photoelectrons?

Ans:

Work function of caesium,  $\Phi_0 = 2.14 \text{ eV}$

Frequency of light,  $\nu = 6.0 \times 10^{14} \text{ Hz}$

(a) The max energy (kinetic) by the photoelectric effect:

$$K = h\nu - \Phi_0$$

Where,

$h =$  Planck's constant  $= 6.626 \times 10^{-34} \text{ Js}$

Therefore,

$$K = \frac{6.626 \times 10^{-34} \times 6 \times 10^{14}}{1.6 \times 10^{-19}} - 2.14$$

$$= 2.485 - 2.140 = 0.345 \text{ eV}$$

Hence, 0.345 eV is the maximum kinetic energy of the emitted electrons.

(b) For stopping potential  $V_0$ , we can write the equation for kinetic energy as:

$$K = eV_0$$

$$\text{Therefore, } V_0 = \frac{K}{e}$$

$$= \frac{0.345 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$= 0.345 \text{ V}$$

Hence, 0.345 V is the stopping potential of the material.

(c) Maximum speed of photoelectrons emitted =  $v$

Following is the kinetic energy relation:

$$K = \frac{1}{2}mv^2$$

Where,

$$m = \text{mass of electron} = 9.1 \times 10^{-31} \text{ Kg}$$

$$v^2 = \frac{2K}{m}$$

$$= \frac{2 \times 0.345 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$$

$$= 0.1104 \times 10^{12}$$

$$\text{Therefore, } v = 3.323 \times 10^5 \text{ m/s} = 332.3 \text{ km/s}$$

Hence, 332.3 km/s is the maximum speed of the emitted photoelectrons.

**Question 11.3: The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?**

**Ans:**

$$\text{Photoelectric cut-off voltage, } V_0 = 1.5 \text{ V}$$

For emitted photoelectrons, the maximum kinetic energy is:

$$K_e = eV_0$$

Where,

$$e = \text{charge on an electron} = 1.6 \times 10^{-19} \text{ C}$$

$$\text{Therefore, } K_e = 1.6 \times 10^{-19} \times 1.5 = 2.4 \times 10^{-19} \text{ J}$$

Therefore,  $2.4 \times 10^{-19}$  J is the maximum kinetic energy emitted by the photoelectrons.

**Question 11.4 Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.**

(a) Find the energy and momentum of each photon in the light beam,

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have a uniform cross-section which is less than the target area), and

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

**Ans:**

$$\text{Monochromatic light having a wavelength, } \lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9} \text{ m}$$

$$\text{Given that the laser emits the power of, } P = 9.42 \text{ mW} = 9.42 \times 10^{-3} \text{ W}$$

$$\text{Planck's constant, } h = 6.626 \times 10^{-34} \text{ Js}$$

$$\text{Speed of light, } c = 3 \times 10^8 \text{ m/s}$$

$$\text{Mass of a hydrogen atom, } m = 1.66 \times 10^{-27} \text{ kg}$$

(a) The photons having the energy as:

$$E = \frac{hc}{\lambda}$$

$\lambda$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}}$$

$$= 3.141 \times 10^{-19} \text{ J}$$

Therefore, each photon has a momentum of :

$$P = \frac{h}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}}$$

$$= 1.047 \times 10^{-27} \text{ kg m/s}$$

(b) Number of photons/second arriving at the target illuminated by the beam = n

Assuming the uniform cross-sectional area of the beam is less than the target area.

Hence, equation for power is written as:

$$P = nE$$

$$\text{Therefore, } n = \frac{P}{E}$$

$$= \frac{9.42 \times 10^{-3}}{3.141 \times 10^{-19}}$$

$$= 3 \times 10^{16} \text{ photons/s}$$

(c) Given that, momentum of the hydrogen atom is equal to the momentum of the photon,

$$P = 1.047 \times 10^{-27} \text{ kg m/s}$$

Momentum is given as:

$$P = mv$$

Where,

v = speed of hydrogen atom

$$\text{Therefore, } v = \frac{P}{m}$$

$$= \frac{1.047 \times 10^{-27}}{1.66 \times 10^{-27}} = 0.621 \text{ m/s}$$

**Question 11.5:** The energy flux of sunlight reaching the surface of the earth is  $1.388 \times 10^3 \text{ W/m}^2$ . How many photons are incident on the Earth per second/square meter? Assume an average wavelength of 550 nm.

**Ans:**

Sunlight reaching the surface of the earth has an energy flux of

$$\phi = 1.388 \times 10^3 \text{ W/m}^2$$

Hence, power of sunlight per square metre,  $P = 1.388 \times 10^3 \text{ W}$

Speed of light,  $c = 3 \times 10^8 \text{ m/s}$

Planck's constant,  $h = 6.626 \times 10^{-34} \text{ Js}$

$\lambda = 550 \text{ nm} = 550 \times 10^{-9} \text{ m}$  is the average wavelength of the photons from the sunlight

Number of photons per square metre incident on earth per second = n

Hence, the equation for power be written as:

$$P = nE$$

$$\text{Therefore, } n = \frac{P}{E}$$

$$= \frac{P\lambda}{hc}$$

$$= \frac{1.388 \times 10^3 \times 550 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^8} = 3.84 \times 10^{21} \text{ photons/m}^2/\text{s}$$

Therefore,  $3.84 \times 10^{21}$  are the photons that are incident on the earth per square meters.

**11.6: In an experiment on the photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be  $4.12 \times 10^{-15}$  V s. Calculate the value of Planck's constant.**

**Ans:**

Given that the slope of cut-off voltage (V) versus frequency ( $\nu$ ) being:

$$\frac{V}{\nu} = 4.12 \times 10^{-15} \text{ Vs}$$

V and frequency being related by the equation as:

$$h\nu = eV$$

Where,

$$e = \text{Charge on an electron} = 1.6 \times 10^{-19} \text{ C}$$

$h$  = Planck's constant

$$\text{Therefore, } h = e \times \frac{V}{\nu}$$

$$= 1.6 \times 10^{-19} \times 4.12 \times 10^{-15} = 6.592 \times 10^{-34} \text{ Js}$$

Therefore,  $6.592 \times 10^{-34}$  Js is the Planck's constant that is determined from the above equation.

**Question 11.7: A 100W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?**

**Ans:**

Power of the sodium lamp  $P = 100\text{W}$

Wavelength of the emitted sodium light,  $\lambda = 589\text{nm}$

$$= 589 \times 10^{-9} \text{ m}$$

Planck's constant,  $h = 6.626 \times 10^{-34} \text{ Js}$

Speed of light,  $c = 3 \times 10^8$

(a)

The energy per photon associated with the sodium light is given as:

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{589 \times 10^{-9}}$$

$$= 3.37 \times 10^{-19} \text{ J} = \frac{3.37 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.11 \text{ eV}$$

(b)

Number of photons delivered to the sphere =  $n$

The equation for power can be written as:

$$P = nE$$

$$\text{Therefore, } n = \frac{P}{E}$$

$$= \frac{100}{3.37 \times 10^{-19}} = 2.96 \times 10^{20} \text{ photons/s}$$

Therefore,  $2.96 \times 10^{20}$  photons are delivered every second to the sphere.

**Question 11.8:** The threshold frequency for a certain metal is  $3.3 \times 10^{14}$  Hz. If the light of frequency  $8.2 \times 10^{14}$  is incident on the metal, predict the cut-off voltage for the photoelectric emission.

**Ans:**

Threshold frequency of the metal,  $\nu_0 = 3.3 \times 10^{14}$  Hz.

Frequency of light incident on the metal,  $\nu = 8.2 \times 10^{14}$  Hz

Charge on an electron,  $e = 1.6 \times 10^{-19}$  C

Planck's constant,  $h = 6.626 \times 10^{-34}$  Js

Cut-off voltage for the photoelectric emission from the metal =  $V_0$

The equation for the cut-off energy is given as:

$$eV_0 = h(\nu - \nu_0)$$

$$V_0 = \frac{h(\nu - \nu_0)}{e}$$

$$= \frac{6.626 \times 10^{-34} \times (8.2 \times 10^{14} - 3.3 \times 10^{14})}{1.6 \times 10^{-19}} = 2.0291 \text{ V}$$

**Question 11.9:** The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

**Ans:**

Work function of the metal,  $\Phi_0 = 4.2 \text{ eV}$

Charge on an electron,  $e = 1.6 \times 10^{-19}$  C

Planck's constant,  $h = 6.626 \times 10^{-34}$  Js

Wavelength of the incident radiation,  $\lambda = 330 \text{ nm} = 330 \times 10^{-9} \text{ m}$

Speed of light,  $c = 3 \times 10^8 \text{ m/s}$

The energy of the incident photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}}$$

$$= 6.0 \times 10^{-19} \text{ J} = \frac{6.0 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.76 \text{ eV}$$

The energy of the incident radiation is less than the work function of the metal. Hence, there is no photoelectric emission taking place.

**Question 11.10:** Light of frequency  $7.21 \times 10^{14}$  Hz is incident in a metal surface. Electrons with a maximum speed of  $6.0 \times 10^5$  m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

**Ans:**

Frequency of the incident photon,  $\nu = 488 \text{ nm} = 488 \times 10^{-9} \text{ m}$

Maximum speed of the electrons,  $v = 6.0 \times 10^5 \text{ m/s}$

Planck's constant,  $h = 6.626 \times 10^{-34} \text{ Js}$

Mass of an electron,  $m = 9.1 \times 10^{-31} \text{ Kg}$

For threshold frequency  $\nu_0$ , the relation for kinetic energy is written as:

$$\frac{1}{2}mv^2 = h(\nu - \nu_0)$$

$$\nu_0 = \nu - \frac{mv^2}{2h}$$

$$= 7.21 \times 10^{14} - \frac{(9.1 \times 10^{-31}) \times (6 \times 10^5)^2}{2 \times (6.626 \times 10^{-34})}$$

$$= 7.21 \times 10^{14} - 2.472 \times 10^{14} = 4.738 \times 10^{14} \text{ Hz}$$

Therefore,  $4.738 \times 10^{14} \text{ Hz}$  is the threshold frequency for the photoemission of the electrons.

**Question 11.11: Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.**

**Ans:**

Wavelength of light produced by the argon laser,

$$\lambda = 488 \text{ nm} = 488 \times 10^{-9} \text{ m}$$

Stopping potential of the photoelectrons,  $V_0 = 0.38 \text{ V}$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$\text{Therefore, } V_0 = \frac{0.38}{1.6 \times 10^{-19}} \text{ eV}$$

Planck's constant,  $h = 6.6 \times 10^{-34} \text{ Js}$

Charge on an electron,  $e = 1.6 \times 10^{-19} \text{ C}$

Speed of light,  $c = 3 \times 10^8 \text{ m/s}$

Using Einstein's photoelectric effect, following is the relation for the work function:

$\Phi_0$  of the material of the emitter as:

$$eV_0 = \frac{hc}{\lambda} - \phi_0 \quad \Phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 488 \times 10^{-9}} - \frac{1.6 \times 10^{-19} \times 0.38}{1.6 \times 10^{-19}}$$

$$= 2.54 - 0.38 = 2.16 \text{ eV}$$

Therefore, 2.16 eV is the work function of the material with which the emitter is made.

**Question 11.12: Calculate the**

**(a) momentum, and**

**(b) the de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.**

**Ans:**

Potential difference,  $V = 56 \text{ V}$

Planck's constant,  $h = 6.6 \times 10^{-34} \text{ Js}$

Mass of an electron,  $m = 9.1 \times 10^{-31} \text{ Kg}$

Charge on an electron,  $e = 1.6 \times 10^{-19} \text{ C}$

**(a)** At equilibrium, the kinetic energy of each electron is equal to the accelerating potential i.e., we can write the relation of velocity ( $v$ ) of each electron as:

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2eV}{m}$$

$$\text{Therefore, } v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 56}{9.1 \times 10^{-31}}}$$

$$= \sqrt{19.69 \times 10^{12}}$$

$$= 4.44 \times 10^6 \text{ m/s}$$

The momentum of each accelerated electron is given as:

$$p = mv$$

$$= 9.1 \times 10^{-31} \times 4.44 \times 10^6 = \mathbf{4.04 \times 10^{-24} \text{ Kg m/s}}$$

Therefore,  $4.04 \times 10^{-24}$  Kg m/s is the momentum of each electron.

(b) de Broglie wavelength of an electron accelerating through a potential V, is given by the relation:

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

$$= \frac{12.27}{\sqrt{56}} \times 10^{-19} \text{ m} = \mathbf{0.1639 \text{ nm}}$$

Therefore, 0.1639 nm is the de Broglie wavelength of each electron.

**Question 11.13: What is the:**

(a) Momentum,

(b) Speed, and

(c) De Broglie wavelength of an electron with a kinetic energy of 120 eV.

**Ans:**

Kinetic energy of the electron,  $E_k = 120 \text{ eV}$

Planck's constant,  $h = 6.6 \times 10^{-34} \text{ Js}$

Mass of an electron,  $m = 9.1 \times 10^{-31} \text{ Kg}$

Charge on an electron,  $e = 1.6 \times 10^{-19} \text{ C}$

(a) For an electron, we can write the relation for kinetic energy as:

$$E_k = \frac{1}{2}mv^2$$

Where, v = speed of the electron

$$\text{Therefore, } v^2 = \sqrt{\frac{2eE_k}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 120}{9.1 \times 10^{-31}}}$$

$$= \sqrt{42.198 \times 10^{12}}$$

$$= 6.496 \times 10^6 \text{ m/s}$$

Momentum of the electron,  $p = mv = 9.1 \times 10^{-31} \times 6.496 \times 10^6$

$$= 5.91 \times 10^{-24} \text{ kg m/s}$$

Therefore,  $5.91 \times 10^{-24}$  Kg m/s is the momentum of the electron.

(b) speed of the electron,  $v = 6.496 \times 10^6 \text{ m/s}$

(c) de Broglie wavelength of an electron having a momentum p, is given as:

$$\lambda = \frac{h}{p}$$

$$= \frac{6.6 \times 10^{-34}}{5.91 \times 10^{-24}} = 1.116 \times 10^{-10} \text{ m} = \mathbf{0.112 \text{ nm}}$$

Therefore, 0.112 nm the de Broglie wavelength of the electron.

**Question 11.14: The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which**

**(a) an electron, and**

**(b) a neutron, would have the same de Broglie wavelength.**

**Ans:**

Wavelength of light of a sodium line,  $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$

Mass of an electron,  $m_e = 9.1 \times 10^{-31} \text{ Kg}$

Mass of a neutron,  $m_n = 1.66 \times 10^{-27} \text{ Kg}$

Planck's constant,  $h = 6.6 \times 10^{-34} \text{ Js}$

**(a)** For the kinetic energy  $K$ , of an electron accelerating with the velocity  $v$ , we have the relation:

$$K = \frac{1}{2} m v^2 \dots\dots\dots (1)$$

We have the relation for de Broglie wavelength as:

$$\lambda = \frac{h}{m_e v}$$

$$\text{Therefore, } v^2 = \frac{h^2}{\lambda^2 m_e^2} \dots\dots\dots (2)$$

**Substituting equation (2) in equation (1), we get the relation:**

$$K = \frac{1}{2} \frac{m_e h^2}{\lambda^2 m_e^2}$$

$$= \frac{h^2}{2 \lambda^2 m_e} \dots\dots\dots (3)$$

$$= \frac{(6.6 \times 10^{-34})^2}{2 \times (589 \times 10^{-9})^2 \times 9.1 \times 10^{-31}}$$

$$= 6.9 \times 10^{-25} \text{ J} = \frac{6.9 \times 10^{-25}}{1.6 \times 10^{-19}} = \mathbf{4.31 \times 10^{-6} \text{ eV}}$$

**Hence, the kinetic energy of the electron is  $6.9 \times 10^{-25} \text{ J}$**

**(b)** Using equation (3), we can write the relation for the kinetic energy of the neutron as:

$$= \frac{h^2}{2 \lambda^2 m_n}$$

$$= \frac{(6.6 \times 10^{-34})^2}{2 \times (589 \times 10^{-9})^2 \times 1.66 \times 10^{-27}}$$

$$= 3.78 \times 10^{-28} \text{ J}$$

$$= \frac{3.78 \times 10^{-28}}{1.6 \times 10^{-19}}$$

$$= 2.36 \times 10^{-9} \text{ eV} = \mathbf{2.36 \text{ neV}}$$

The neutron has the kinetic energy of  $3.78 \times 10^{-28} \text{ J}$  or  $2.36 \text{ neV}$ .

**Question 11.15: What is the de Broglie wavelength of:**

- (a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,
- (b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and
- (c) a dust particle of mass  $1.0 \times 10^{-9}$  kg drifting with a speed of 2.2 m/s?

**Ans:**

(a) Mass of the bullet,  $m = 0.040$  Kg

Speed of the bullet,  $v = 1.0$  km/s = 1000 m/s

Planck's constant,  $h = 6.6 \times 10^{-34}$  Js

de Broglie wavelength of the bullet is given by the relation :

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{0.040 \times 1000}$$

$$= 1.65 \times 10^{-35} \text{ m}$$

(b) Mass of the ball,  $m = 0.060$  Kg

Speed of the ball,  $v = 1.0$  m/s

de Broglie wavelength of the ball is given by the relation:

$$= \lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{0.060 \times 1}$$

$$= 1.1 \times 10^{-32} \text{ m}$$

(c) Mass of the dust particle,  $m = 1 \times 10^{-9}$  Kg

speed of the dust particle,  $v = 2.2$  m/s

de Broglie wavelength of the dust particle is given by the relation:

$$= \lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{2.2 \times 1 \times 10^{-9}}$$

$$= 3.0 \times 10^{-25} \text{ m}$$

**Question 11.16: An electron and a photon each have a wavelength of 1.00 nm. Find:**

- (a) Their momenta,
- (b) The energy of the photon, and
- (c) The kinetic energy of the electron.

**Ans:**

Wavelength of an electron  $\lambda_e$  and a photon  $\lambda_p$  ,

$$\lambda_e = \lambda_p = \lambda = 1 \text{ nm} = 1 \times 10^{-9} \text{ m}$$

Planck's constant,  $h = 6.63 \times 10^{-34}$  Js

(a) The momentum of an elementary particle is given by de Broglie relation:

$$\lambda = \frac{h}{p} \quad p = \frac{h}{\lambda}$$

It is clear that momentum depends only on the wavelength of the particle. Since the wavelengths of an electron and a photon are equal, both have an equal momentum.

$$\text{Therefore, } p = \frac{6.63 \times 10^{-34}}{1 \times 10^{-9}}$$

$$= 6.63 \times 10^{-25} \text{ Kg m/s}$$

(b) The energy of a photon is given by the relation:

$$E = \frac{hc}{\lambda}$$

Where,

Speed of light,  $c = 3 \times 10^8 \text{ m/s}$

$$\text{Therefore, } E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1 \times 10^{-9} \times 1.6 \times 10^{-19}}$$

$$= 1243.1 \text{ eV} = 1.243 \text{ keV}$$

**Therefore, the energy of the photon is 1.243 keV.**

(c) The kinetic energy (K) of an electron having momentum p, is given by the relation:

$$K = \frac{1}{2} \frac{p^2}{m}$$

Where, m = Mass of the electron =  $9.1 \times 10^{-31} \text{ Kg}$

$$p = 6.63 \times 10^{-25} \text{ Kg m/s}$$

$$\text{Therefore, } K = \frac{1}{2} \times \frac{(6.63 \times 10^{-25})^2}{9.1 \times 10^{-31}}$$

$$= 2.415 \times 10^{-19} \text{ J}$$

$$= \frac{2.415 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.51 \text{ eV}$$

1.51eV is the kinetic energy of the electron.

**Question 11.17:**

(a) For what kinetic energy of a neutron will the associated de Broglie wavelength be  $1.40 \times 10^{-10} \text{ m}$  ?

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of  $(3/2) \text{ kT}$  at 300 K.

**Ans:**

(a) de Broglie wavelength of the neutron,  $\lambda = 1.40 \times 10^{-10} \text{ m}$

Mass of a neutron,  $m_n = 1.66 \times 10^{-27} \text{ Kg}$

Planck's constant,  $h = 6.6 \times 10^{-34} \text{ Js}$

Kinetic energy (K) and velocity (v) are related as:

$$K = \frac{1}{2} m_n v^2 \dots\dots\dots (1)$$

de Broglie wavelength ( $\lambda$ ) and velocity (v) are related as:

$$\lambda = \frac{h}{m_n v} \dots\dots\dots (2)$$

Using equation (2) and equation (1), we get:

$$K = \frac{1}{2} \frac{m_n h^2}{\lambda^2 m_n^2}$$

$$= \frac{h^2}{2\lambda^2 m_n}$$

$$= \frac{(6.63 \times 10^{-34})^2}{2 \times (1.40 \times 10^{-10})^2 \times 1.66 \times 10^{-27}}$$

$$= 6.75 \times 10^{-21} \text{ J}$$

$$= \frac{6.75 \times 10^{-21}}{1.6 \times 10^{-19}}$$

$$= 4.219 \times 10^{-2} \text{ eV}$$

Hence, the kinetic energy of the neutron is  $6.75 \times 10^{-21} \text{ J}$  or  $4.219 \times 10^{-2} \text{ eV}$ .

(b) Temperature of the neutron,  $T = 300\text{K}$

**Boltzmann constant,  $k = 1.38 \times 10^{-23} \text{ Kg m}^2 \text{ s}^{-2} \text{ K}^{-1}$**

Average kinetic energy of the neutron:

$$K' = \frac{3}{2} kT$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$$

$$= 6.21 \times 10^{-21} \text{ J}$$

The relation for the de Broglie wavelength is given as:

$$\lambda' = \frac{h}{\sqrt{2K'm_n}}$$

Where,

$$m_n = 1.66 \times 10^{-27} \text{ Kg}$$

$$h = 6.6 \times 10^{-34} \text{ Js}$$

$$K' = 6.75 \times 10^{-21} \text{ J}$$

$$\text{Therefore, } \lambda' = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.21 \times 10^{-21} \times 1.66 \times 10^{-27}}}$$

$$= 1.46 \times 10^{-10} \text{ m} = 0.146 \text{ nm}$$

Therefore, 0.146nm is the de Broglie wavelength of the neutron.

**Question 11.18: Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).**

**Ans:**

The momentum of a photon having energy ( $h\nu$ ) is given as:

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \quad \lambda = \frac{h}{p} \dots\dots\dots (i)$$

Where,

$\lambda$  = wavelength of the electromagnetic radiation

**c = speed of light**

**h = Planck's constant**

De Broglie wavelength of the photon is given as:

$$\lambda = \frac{h}{mv}$$

But, **p = mv**

$$\text{Therefore, } \lambda = \frac{h}{p} \dots\dots\dots(ii)$$

Where,  $m$  = mass of the photon

$v$  = velocity of the photon

From equation (i) and (ii) it can be concluded that the wavelength of the electromagnetic radiation and the de Broglie wavelength of the photon are equal.

**Question 11.19: What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean-square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)**

**Ans:**

Temperature of the nitrogen molecule,  $T = 300 \text{ K}$

Atomic mass of nitrogen = **14.0076 u**

Hence, mass of the nitrogen molecule,  $m = 2 \times 14.0076 = 28.0152 \text{ u}$

But,  $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$

Therefore,  $m = 28.0152 \times 1.66 \times 10^{-27} \text{ kg}$

**Planck's constant,  $h = 6.63 \times 10^{-34} \text{ Js}$**

**Boltzmann constant,  $k = 1.38 \times 10^{-23} \text{ J/K}$**

We have the expression that relates mean kinetic energy ( $\frac{3}{2}kT$ ) of the nitrogen molecule with the root mean square speed ( $V_{\text{rms}}$ ) as:

$$\frac{1}{2}mv_{\text{rms}}^2 = \left(\frac{3}{2}kT\right)$$

$$V_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

For nitrogen molecule, the de Broglie wavelength is given as:

$$\lambda = \frac{h}{mv_{\text{rms}}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 28.0152 \times 1.66 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$= 0.028 \times 10^{-9} \text{ m} = \mathbf{0.028 \text{ nm}}$$

Therefore, the nitrogen molecule is 0.028 nm is the de Broglie wavelength.